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# Linear Optimization and Extensions: Theory and Algorithms


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*Dedicated to our families:*
*Chi-Hsin Chao Fang*
*Mini and Vidya Puthenpura*

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# Preface

Since G. B. Dantzig first proposed the celebrated simplex method around 1947, the wide applicability of linear programming models and the evolving mathematical theory and computational methodology under these models have attracted an immense amount of interest from both practitioners and academicians. In particular, in 1979, L. G. Khachian proved that the ellipsoid method of N. Z. Shor, D. B. Yudin, and A. S. Nemirovskii could outperform the simplex method in theory by exhibiting polynomial-time performance; and, in 1984, N. Karmarkar designed a polynomial-time interior-point algorithm that rivals the simplex method even in practice. These three methods present different and yet fundamental approaches to solving linear optimization problems.

This book provides a unified view that treats the simplex, ellipsoid, and interior-point methods in an integrated manner. It is written primarily as a textbook for those graduate students who are interested in learning state-of-the-art techniques in the area of linear programming and its natural extensions. In addition, the authors hope it will serve as a useful handbook for people who pursue research and development activities in the relatively new field of interior-point methods for optimization.

We have organized the book into ten chapters. In the first chapter, we introduce the linear programming problems with modeling examples and provide a short review of the history of linear programming. In the second chapter, basic terminologies are defined to build the fundamental theory of linear programming and to form a geometric interpretation of the underlying optimization process. The third chapter covers the classical simplex method—in particular, the revised simplex method. Duality theory, the dual simplex method, the primal-dual method, and sensitivity analysis are the topics of Chapter 4. In the fifth chapter, we look into the concept of computational complexity and show that the simplex method, in the worst-case analysis, exhibits exponential complexity. Hence the ellipsoid method is introduced as the first polynomial-time algorithm for linear programming. From this point onward, we focus on the nonsimplex approaches. Naturally, the sixth chapter is centered around the recent advances of Karmarkar's algorithm and its polynomial-time solvability. Chapter 7 essentially covers the affine scaling variants, including the primal, dual, and primal-dual algorithms, of Karmarkar's method. The concepts of central trajectory and path-following are also included. The eighth chapter reveals the insights of interior-point methods from both the algebraic and geometric viewpoints. It provides a platform for the comparison of different interior-point algorithms and the creation of new algorithms. In Chapter 9, we extend the results of interior-point-based linear programming techniques to quadratic and convex optimization problems with linear constraints. The important implementation issues for computer programming are addressed in the last chapter. Without understanding these issues, it is impossible to have serious software development that achieves the expected computational performance.

The authors see three key elements in mastering linear optimization and its extensions, namely, (1) the intuitions generated by geometric interpretation, (2) the properties proven by algebraic expressions, and (3) the algorithms validated by computer implementation; and the book is written with emphasis on both theory and algorithms. Hence it is implied that a user of this book should have some basic understanding in mathematical analysis, linear algebra, and numerical methods. Since an ample number of good reference books are available in the market, we decided not to include additional mathematical preliminaries.

This book pays special attention to the practical implementation of algorithms. Time has proven that the practical value of an algorithm, and hence its importance among practitioners, is largely determined by its numerical performance including robustness, convergence rate, and ease of computer implementation. With the advent of digital computer technology, iterative solution methods for optimization have become extremely popular. Actually, this book explains various algorithms in the framework of an iterative scheme with three principal aspects: (a) how to obtain a starting solution, (b) how to check if a current solution is optimal, and (c) how to move to an improved solution. We have attempted to cast all the algorithms discussed in the book within the purview of this philosophy. In this manner, computer implementation follows naturally.

The material in this book has been used by the authors to teach several graduate courses at North Carolina State University, University of Pennsylvania, and Rutgers University since 1988. According to our experience, Chapters 1 through 6 together with a brief touch of Chapter 7 comprise the material for a one-semester first graduate course in Linear Programming. A review of Chapters 3 and 5 together with Chapters 6 through 10 could serve for another one-semester course in Advanced Linear Programming, or Special Topics on Interior-Point Methods. This book can also be used as a "cookbook" for computer implementation of various optimization algorithms, without actually going deep into the theoretical aspects. For this purpose, after introducing each algorithm, we have included a step-by-step implementation recipe.

We have tried to incorporate the most salient results on the subject matter into this book. Despite our efforts, however, owing to the tremendous ongoing research activities



in the field of interior-point methods, we may have unintentionally left out some of the important and recent work in the area.

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