OR/MA 706: NONLINEAR PROGRAMMING

A Graduate Level Course

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North Carolina State University – Fall 2024

Course page information

- https://www.ise.ncsu.edu/fuzzy-neural/home/courses/or-ma-st706-nonlinear-programming/

- Syllabus
- References
- Lecture Notes
- Supplemental Reading Materials
- Homework Assignments (6)
- Hand-on Exercises (2)
- Exams (2)
- Course Grades

Teaching information

- **Instructor**

  Professor S.-C. Fang (fang@ncsu.edu)
- **Office**

  4341 Fitts-Woolard Hall <919.515.2192>
- **Meeting Time**

  Tu, Th 1:30 PM – 2:45 PM
- **Classroom**

  FWH 4141
- **Office Hours**

  Tu, Th 3:00 PM – 4:00 PM (or by online appointment)
- **Teaching Assistant**

  Lesheng Wang (lwang65@ncsu.edu)
- **Office Hours**

  Mon, Wed 11:30 AM – 12:30 PM at FWH 4333

Course information

- Course objective
- Pre-requisites
- Course content
- Grading and exams
- Classroom rules
- Textbook and references

Course objective

- OR/MA 706 is a graduate-level course that prepares students to understand the theory and algorithms of nonlinear optimization.
- It involves mathematical analysis, algorithm design and numerical methods.
- It is a preparatory course for OR students to take their PhD Qualifying Exams on this subject.
- It intends to get students exposed to basic machine learning and artificial intelligence.

Important to know

- This course is designed mainly for PhD students with proper background, research interests and self-learning capabilities.

- This course does NOT teach you how to concretely model a system problem.

- This course does NOT tell you how to manage existing data banks.

- This course does NOT tell you how to use Excel, SAS OPT, MATLAB, Python, LINGO, CPLEX, CVX, or any commercial solver and software platform.

Important to know

- This course INTENDS to prepare you to understand the mathematical theory and solution methods of optimization problems involving nonlinear functions in the objective function or constraints.

- This course INTENDS to prepare you for reading existing literatures.

- This course INTENDS to prepare you for conducting research in optimization with applications to machine learning and artificial intelligence.

Prerequisites

1. ISE/OR/MA 505: Linear Programming
2. (Self-learning) Programming using CPlex, Gurobi, CVX, SeDuMi on MATLAB or Python.

Course contents

I. Introduction

II. Unconstrained Optimization

- Motivation, Intuition, Speculation and Theorization
- Basic Properties and Optimality Conditions
  - First order information
  - Second order information
- Solution Methods

III. Constrained Optimization

- Basic Properties and KKT Optimality Conditions
- Lagrange Dual Problem
- Sensitivity Analysis
- Solution Methods

Course contents

IV. Applications to Machine Learning

- Multi-layer Neural networks (NN) for deep learning
- Support vector machines (SVM) for supervised learning
- Support vector regression (SVR)
- Clustering for unsupervised learning

V. Extended Topics

- Semidefinite Programming (SDP)
- Second Order Cone Programming (SOCP)

Machine learning and optimization

1. Many machine learning problems are formulated as minimization of some loss function that measures discrepancy between the predictions of the model being trained and the actual problem instances, or as maximization of some reward function that affirms an expected decision.

2. One major difference between machine learning and optimization lies in the goal of generalization - optimization intends to minimize the loss/maximize the reward on a set of seen examples while machine learning is concerned with minimizing the loss/maximizing the reward on unseen samples.

Basic approaches of machine learning

I. Supervised learning for classification and prediction

- Support Vector Machines & Regression (SVM & SVR)
- Artificial Neural Networks (ANN)

II. Unsupervised learning for clustering and featuring

- Similarity Learning and Sparse Optimization

III. Reinforcement learning in dynamic environment

- Markov Decision Process and Dynamic Programming

Homework / project / exam

- Homework assignments (6)
  - weekly or biweekly
  - individual
- Hand-on exercises on machine learning (2)
  - report and presentation
  - at most 2 persons a team
- Exams (2)
  - Unconstrained optimization
  - Constrained optimization

Grading

1. Homework assignments - 30%
2. Hand-on exercises - 30%
3. Exam I - 20%

   Exam II – 20%

A - 85 and above

B - 70 to 84

C - 60 to 69

Fail - under 60

Classroom rules

- Rule 1: No late homework without TA’s pre-approval.
- Rule 2: Turn in your homework through email to TA and copy to Dr. Fang.
- Rule 3: Convince TA for grade changes.
- Rule 4: No make-up exam without pre-approval or an official "doctor’s note“.

Textbook and references

Textbook:

- D. G. Luenberger and Y. Ye, "Linear and Nonlinear Programming," 4th Edition, 2016, Springer, ISBN 978-3-319-18842-3 (eBook)

Textbook and references

References: (Classical)

1. R.T. Rockafellar : Convex Analysis,

Princeton University Press, ISBN: 0691080690, 1970.

2. P.E. Gill, W. Murray, M.H. Wright : Practical Optimization, QA402.5 .G54, 1981.

3. G.P. McCormick : Nonlinear Programming : Theory, Algorithms and Applications, T57.8 .M39, 1983.

4. M.S. Bazaraa, H.D. Sherali and C.M. Shetty : Nonlinear Programming : Theory and Algorithms, T57.8 .B39, 1993.

Textbook and references

References: (More recent)

5. S. Boyd and L. Vandenberghe, Convex Optimization, 7th Printing, Cambridge University Press, ISBN 0-521-83378-7, 2009.

6. A. Nemirovski : Lectures on Modern Convex Optimization, ISYE, Georgia Tech., 2005.

7. E. G. Birgin and J. M. Martinez, "Practical Augmented Lagrangian Methods for Consttrained Optimization," Society of Industrial and Applied Mathematics, ISBN 978-1-611973-35-8, 2014.

Textbook and references

References: (Extended topics)

8. C.J. Goh, X.Q. Yang : Duality in Optimization and Variational Inequalities, ISBN: 0415274796, 2002.

9. F. Facchinei, J.-S. Pang : Finite-Dimensional Variational Inequalities and Complementarity Problems, ISBN: 038795581X, 2003.

10. S.-C. Fang, W. Xing: Linear Conic Programming, Science Press, ISBN: 9787030381767, 2013.

Textbook and references

References: (Machine learning)

11. A. Messac, Optimization in Practice with Matlab, Cambridge Univ. Press, ISBN: 9781107109186, 2015.

12. P. Flach, Machine Learning, Cambridge University Press, ISBN 978-0-511973-00-0, 2012.

Historical papers：

1. Karush W. 1939. Minima of functions of several variables with inequalities as side constraints. Master's thesis, Dept. of Mathematics, Univ. of Chicago.

2. Kuhn H.W. Tucker A.W. 1951. Nonlinear programming. In: Proc. 2nd Berkeley Symposium on Mathematical Statistics and Probabilistics, Berkeley. University of California Press, pp. 481–492.

Some thoughts on artificial intelligence

1. The main goal of artificial intelligence (AI) is to equip machines (computers) with human (or super-human, hopefully) intelligence for dynamic decision making.

2. Computers examine data to extract embedded information (data mining) to form useful knowledge (machine leaning) for right decision making (analytics).

3. Algorithms guide machines to perform each desired task step by step.

Systems analytics

- Modern information technology (IT) provides industries with the ability to collect vast amounts of data (Big Data) of underlying business systems.

- Data requires more than just flashy dashboards and reports. Information and knowledge (Data Mining and Machine Learning) are the key.

- Systems analytics help us make better data-driven business decisions for industries (Analytic DM).

Operations research - SAO

- OR models can be useful for machine learning and analytic decision making with dynamic or transactional data (AI).

- SAO provides mathematical/analytic theory and tools for modeling system problems of interests to make optimal decisions using dynamic big data.

New initiatives of SAO area

- A sequence of four courses on systems analytics

1. Optimization models for systems analytics
2. Stochastic models for systems analytics
3. Statistical models for systems analytics
4. Simulation models for systems analytics