

- (10 points) Write a short essay (in 250 words) to tell me something about you - (a) Why do you want to take this course? and (b) What do you expect to learn from this course? Please submit your essay by email to TA using your preferred email address.
- (10 points) Provide 10 real-valued nonlinear functions of different types defined on the one-dimensional Euclidean space E^1 .
- (24 points) Compute the following limits (whenever a limit exists):

$$(a) \lim_{\substack{(x, y) \rightarrow (0, 0) \\ (x, y) \in S}} \frac{xy}{x^2 + y^2}, \quad (b) \lim_{\substack{(x, y) \rightarrow (0, 0) \\ (x, y) \in S}} \frac{x^2 - y^2}{x^2 + y^2},$$

where S can be the following sets in E^2 :

- $S = \{(x, y) \mid y = ax\} \quad (a \neq 0)$.
 - $S = \{(x, y) \mid y = ax^2\} \quad (a \neq 0)$.
 - $S = \{(x, y) \mid y^2 = ax\} \quad (a \neq 0)$.
- (18 points) Find the first-order information (Gradient function) and second-order information (Hessian function) of the following real-valued functions defined on E^2 :

(a) $f(x, y) = 5x^2 + 25xy + 10y^2 + 100$

(b) $f(x, y) = \frac{xy}{x^2 + y^2 + 1}$.

(c) $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2 - 1}$.

- (24 points) In each of the following cases, determine the intervals in which the function f is increasing or decreasing and find the maximum and minimum solutions (if any) in the set where each f is defined. (A graph of each function may help you answer the questions.)

(a) $f(x) = x^3 + ax + b, \quad x \in E^1$.

(b) $f(x) = \log(x^2 - 9), \quad |x| > 3$.

(c) $f(x) = x^{2/3}(x - 1)^4, \quad 0 \leq x \leq 1$.

(d) $f(x) = \begin{cases} (\sin x)/x, & \text{if } x \neq 0 \\ 0, & \text{otherwise} \end{cases}, \quad 0 \leq x \leq \pi/2$.

6. (14 points) Let f and g be two real-valued functions defined on E^1 with $f(x) \neq 0$, $g(x) \neq 0$, $f'(x) \neq 0$ and $g'(x) \neq 0$, for all x in E^1 . If $f(x)g(x) = 1$ for all x , then find the values of the following two expressions:

(a) $f'(x)/f(x) + g'(x)/g(x) = ?$

(b) $f''(x)/f'(x) - 2f'(x)/f(x) - g''(x)/g'(x) = ?$