Issued: 09/15/2024 **OR/MA 706 HW#2** Due: Tuesday 09/24/2024 in Class

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Reading Assignment: Please read Appendix A and Appendix B of Luenberger and Ye's textbook before doing your homework.

Exercise:

1. (3 pts x 6 = 18 pts) Find all the interior and accumulation points of the following sets in E^1 and decide whether each set is open, closed, or neither.
   (a) S is the set of all integers.
   (b) S is the set of all numbers in the form of $1/n$, for $n = 1, 2, 3, \cdots$.
   (c) S is the interval $(a, b]$ with $a < b$.
   (d) S is the union of interval (a, b) and interval (c, d) with $a < b$ and $c < d$.
   (e) S in the intersection of interval $[a, b]$ and interval $[c, d]$ with $a < b$ and $c < d$.
   (f) S is the set of all numbers in the form of $2^{-n} + 5^{-m}$, for $m, n = 1, 2, \cdots$.

2. (5 pts x 5 = 25 pts) Prove the following statements:
   (a) Let $I = \{1, 2, ..., p\}$ be an index set. If C_i is a convex set in E^n for every $i \in I$, then the intersection of this collection of convex sets is a convex set.
   (b) The closure of a convex set is a convex set.
   (c) The Cartesian product $C_1 \times C_2 = \{(x, y) \in E^{2n} \mid x \in C_1, y \in C_2\}$ of two convex sets C_1 and C_2 is a convex set in E^{2n}.
   (d) The vector sum $C_1 + C_2 = \{z \in E^n \mid z = x + y \;\; with \;\; x \in C_1, y \in C_2\}$ of two convex sets C_1 and C_2 is a convex set.
   (e) The image of a convex set $C \in E^n$ under a linear function/mapping $f : E^n \to R$ is a convex set $f(C) \in R$.

3. (10 pts x 3 = 30 pts)

   I know you are smart enough to figure out all optimal solutions to the three problems listed below. But I want you to go through, step by step, the first-order and second-order necessary and sufficient conditions to identify all optimal solutions of each problem.

a.

$$\begin{array}{ll} \text{Min} & \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 \\ \text{s. t.} & \\ & -2 \le x_1 \le 2 \\ & -2 \le x_2 \le 2 \end{array}$$

b.

$$\begin{array}{ll} \text{Min} & -\frac{1}{2}x_1^2 - \frac{1}{2}x_2^2 \\ \text{s. t.} & \\ & -1 \le x_1 \le 2 \\ & -1 \le x_2 \le 1 \end{array}$$

c.

$$\begin{array}{ll} \text{Min} & \frac{1}{2}x_1^2 - \frac{1}{2}x_2^2 \\ \text{s. t.} & \\ & -1 \le x_1 \le 1 \\ & -2 \le x_2 \le 1 \end{array}$$

4. (10 pts x 3 = 30 pts)

   It is the time for you to get familiar with at least one software that may help you learn the course material and do your homework and projects.

   Please use either Matlab, Python or any other software to draw the objective function of the above three problems over their feasible domains and verify your solutions are correct.