OR/MA/ST 706 HW #6

Issue Date: Nov. 19, 2024 Due: Noon, Nov. 27, 2024

Problem 1 (50 points): Consider the following problem:

$$\begin{aligned} Minimize \quad & x_1 \\ \text{s.t.} \quad & x_1^2 + x_2^2 = 1 \end{aligned}$$

(1) (10 points) Use the KKT conditions to find the optimal solutions to this primal problem, step by step.
(2) (10 points) Derive its Lagrangian dual problem, step by step.
(3) (5 points) Verify that the dual objective function is concave.
(4) (10 points) Find the optimal solution(s) to the dual problem, step by step.
(5) (5 points) Is the primal problem a convex optimization problem? Why? Is there any duality gap? Why?
(6) (10 points) If we increase the right-hand-side value of the constraint by 0.00001, use the sensitivity theory to estimate the optimal objective value of the new problem

Problem 2 (50 points): Consider the following problem:

$$\begin{aligned} Minimize \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 1 \end{aligned}$$

(1) (10 points) Use KKT conditions to find the optimal solutions to this primal problem, step by step.
(2) (10 points) Derive its Lagrangian dual problem, step by step.
(3) (5 points) Verify that the dual objective function is concave.
(4) (10 points) Find the optimal solution(s) to the dual problem, step by step.
(5) (5 points) Is there any duality gap? Why?
(6) (10 points) If we decrease the right-hand-side value of the constraint by 0.00001, use the sensitivity theory to estimate the optimal objective value of the new problem